[Alta. Dept. of Agriculture. Water Resources Division]

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Application of Critical Path Method For
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APPLICATION
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CRITICAL PATH METHOD
FOR
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APPLICATION
OF
CRITICAL PATH METHOD
FOR
WATER RESOURCES PROJECTS

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INTRODUCTION

NOTES

Since the successful application of Critical Path Method (CPM) in the National Aeronautics and Space Administration and initial application in chemical and construction industries in the 1950's, the use of CPM has grown at a rapid rate. Articles in professional and trade journals have amounted to at least 500 articles in the past few years and separate journals continously carry advertisements for public training courses on the subject of CPM. More recently, Water Resources Journal, a U.S.A. Publication recommends application of CPM in Water Resources Projects. Since the U.S.A. Department of Navy Office first required the use of PERT (Program Evaluations and Review Technique) and CPM by contractors on its large programs, the number of agencies having similar requirements have been increased each year. Almost every major research and development program requires the prime contractor and major contractors to utilize some form of network diagram and Critical Path.

CPM has also been introduced in the contract specifications by architects, engineers, the State Highway Departments, the university building authorities and utility companies. In addition manufacturing companies are applying the Critical Path approach to large maintenance projects, equipment modifications and its installation, sales promotion programs and a variety of other types of projects.

The following are a few agencies that use Critical Path Method in their projects:

Special Projects Office, Department of Navy.

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Department of Air Force, Department of Army, Corps of Engineers, National Aeronautics and Space Administration, Federal Aviation Agency, and National Defense of Canada, and many other private companies.

The use of Critical Path ensures the most economical planning of operations in order to fulfill desirable completion dates for projects. It supplements the personal judgement and experience or trial and error formerly used to select operation times, size of equipment, required resources, etc. Indeed with Critical Path, one readily determines with certainty the best completion date for the project and finally it provides the means of assessing the effect of all variations, i.e. change of orders, extra work or reductions upon the time of completion and often the cost of the work. In the past, a rational basis of calculating such effects was seriously lacking and led to many unpleasant outcomes. Today the time and cost of every change from the original optimum plan can be rapidly assessed at any stage of the planning, scheduling or construction of the project. CPM technique is already well established in a variety of industries and particularly suited to building and construction works, and planning of major projects. It is therefore of major importance to contractors, engineers, economists, agrologists, estimators, work managers, and clerical help. It is also vital to others whose work is related to construction, planning, scheduling, financing, cost estimating and work management.

NOTES

Some contract documents already are specifying that the contractor should submit a critical path diagram at the commencement of work and several contractors have been using critical path as part of their project planning. Increasing the use of Critical Path will demonstrate the importance of time implements, records and cost performance and information for the construction industry.

Critical Path could be used on any project of any size, in any location. There is no limitation to its application.

A project must be defined, planned, scheduled, and controlled in order to achieve its objective or objectives. CPM is one of the best aids to planning, scheduling and controlling of projects.

NOTES

ADVANTAGES OF CRITICAL PATH METHOD

1. Plans and defines projects and their objectives.
2. Prevents overlapping of work.
3. Saves time and resources by preventing misallocation of time and resources.
4. Provides better communication between various individuals or organizations involved, and better understanding of individual responsibilities.
5. Formulates the optimal starting dates and estimated completion dates of projects.
6. Eliminates confusion by providing a starting point for each phase of a project.
7. Eliminates rush jobs by proper scheduling.
8. Creates confidence while carrying out a project.
9. Provides correct answers for government officials and for the public.
10. Assists in requests for more staff and more funds by presenting a concise, logical development program.
11. The experience could be useful for other agencies.
12. Encourages a disciplined approach in the planning, scheduling, and control of projects.
13. Provides a standard method for documentation, communication of plans, scheduling and evaluation of time and cost performances.
14. Identifies the most critical elements in a plan, and focuses managements' attention on any constraining phase of the project.

15. Permits computations which subsequently permit activities to be carried either advanced or delayed without affecting completion time of the project.
16. Determines an acceptable method of eliminating the undesirable demands on the resources.

CRITICAL PATH

The Critical Path Method is a new and powerful tool for the planning, managing and scheduling of all types of projects. It is a graphical representation of the operations, involved in achieving the objective or objectives of a project. It could be applied to any Water Resources project in which we are interested. It could easily answer the following questions:

1. What is to be done?
2. How is it to be done?
3. By whom is it to be done?
4. When is it to be done?
5. How much will it cost?

The objectives of a project must be defined in order to prepare network diagrams. This is the answer for what. The alternatives and operations to be performed to achieve the objectives of the project, are also shown in the diagram. This is the answer for How. It could also show the allocation of facilities and equipment. The network diagram and consequently the critical path method (CPM) will provide information such as designating the individuals or groups which should perform each operation. In other words, it allocates the human resources. This will answer the question of By whom.

The most important task is to estimate the required time and cost for each project. These will answer the questions of When and How much.

A fully developed critical path network is a logical mathematical model of the project to be used in the planning, scheduling and the management of any project.

In all projects, we are to a greater or lesser degree concerned with developing an optimal, or at least a workable plan of the activities which make up the project. We are concerned with the manpower and facilities required to carry out the program. In brief, we are interested in first; planning the work and then working the plan.

NETWORK DIAGRAM:

The network flow diagram is a graphic presentation of operations involved in a project. This diagram shows the start, end, and correct relationship between any two and all operations. Each operation (activity) in a project is related to another operation (activity) in the project in one or more of the following ways:

1. What activities (operation) must precede this activity?
2. What activities must follow this activity?
3. What activities may be performed concurrently with this activity?

The above are logical sequential questions which must be answered for each activity. The network diagram is the basic step for CPM.

BASIC TERMS

NOTES

Activity:

An activity is any portion of a project having a definite beginning and ending. An activity in CPM may represent labor, professionals, personnel, machinery operations, paper work or any other resource which consumes time or money as a portion of a project. For example, in our work, an activity could be Hydrology study, economic study, soil classifications, design, investigation, etc. Synonymous with "activity" is "task", "job", "operations", and "job element".

Dummy:

A dummy is a dotted arrow representing merely a dependency among operations (activities). It does not identify any magnitude or quantity, such as time, money or any other resources.

Event:

The beginning and ending points of activities are called "events". Synonyms are "Node" and "Connector".

Merge Point:

If an event represents the joint completion of more than one activity it is called a merge point.

"Burst Point Entry"

If an event represents the joint initiation of more than one activity, it is called "burst point event".

Table No. 1 shows some of the symbols used in network diagrams.

TABLE NO. 1

Term	Symbol	Name of Symbol
Activity	⟶	Arrow
Event	○ or ▯ or □	Node
Dummy	- - - - - - - - - - →	Dummy

DEFINITION OF PROJECT

Prior to outlining how network analysis and critical paths are applied in project management, it is necessary to consider some additional basic concepts and definitions of the project. A project is any task which has a definable beginning and a definable end. It requires expenditure of one or more resources in each of the separate activities, which can be interrelated or independent. Each activity must be completed to achieve the objectives for which the task was instituted.

ELEMENTS OF PROJECT

There are three overall categories of elements:

1) Operations or the things we do.
2) Resources or the things we use.
3) The conditions or restraints under which we must work.

Categories 2 and 3 are generally outside of our control, whereas part of category one (1) are under our control.

Resource elements that we use include; men, materials, machines, money and time. It is possible that we may have internal restraints such as not enough personnel or money.

NOTES

The objective is to co-ordinate all elements (activities) by construction of a master plan (network diagram) which will become a working model of the project.

NETWORK DIAGRAM PREPARATION:

The first step necessary for construction of the network diagram is the definition of all elements or activities required to complete the project and includes the following steps:

1. Definition of project.
2. Definition of project objectives.
3. What jobs ought to be done.
4. Sequence of job performance.

   The degree of breakdown will vary for each project and will be influenced by the nature of the job, class of labor involved, costing data required, management, government and resources. A good example is the General Network Diagram prepared for Water Resources Development projects.
5. Prepare a list of activities from step 3.
6. Determine the logical relationships between the activities. This is one of the most important steps and should be carefully done. This step is best done by an experienced person having adequate technical knowledge of the activities under examination. Physical constraints applicable to activities should be determined. For example, drilling for foundation; investigation of a dam site could not be started unless the center line

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of the dam and the test hole locations are mapped and the drilling equipment and men are ready to work.

The following sequential questions should be answered for each activity:

1. What activities must precede this activity?
2. What activities must follow this activity?
3. What activities may be done concurrently with this activity?

In this way each activity is examined and necessary sequences of activities are determined. The end of one activity may signal the start of a related and dependent activity. Consequently, overlapping operations are omitted. With the Critical Path Method, overlapping is impossible.

There are various constraints which also have to be discussed at this stage. Some of these constraints are:

1. Physical constraints, as previously mentioned.
2. Safety constraints. For example: Flying for an inspection on a snowy day.
3. Equipment constraints. Release of drills from another project.
4. Management constraints. Decision of management could control the start of a job.
5. Fund constraint.

These constraints may necessitate the sequential separation of activities which could otherwise be concurrent. All these aspects must therefore be carefully studied by the planner when the project is formulated and broken down.

BASIC RULES OF NETWORK DIAGRAM

NOTES

1. Before an activity may begin, all activities preceding it must be complete.
2. Arrows imply logical precedence only. According to one author, neither the length of the arrow, nor its direction on the drawing has any significance, but this writer believes that the direction is important and indicates the direction of progress in the project. There are some network diagrams prepared with time scale, but unless it is mentioned, the length of arrow has no significance.

USE OF COMPUTERS

When there is considerable complexity in project sequences, all the network calculations have to be done by computers. This necessitates the development of suitable computer programming, data processing and punch card operation. Large engineering projects may require computer processing because of the vast number of activities involved since it has been found that many computers have already established programs that will analyse as many as 1500 to 2000 activities in a few hours from the time the data has been set up for this machine. For a small number of activities, manual methods could be applied.

LEAD TIME & END TIME

The completed network diagram should have only one point of entry and one exit. In order that this be so, the first activity is designated lead time. A job which by definition must precede all other activities. These times may correspond to real durations; the lead time being the interval

between receiving final approval of the project and actually starting work on it. In other words, "get ready". The end time being the interval between actual and scheduled completion times may correspond to the activity, "clear away". Both may be much the same as a dummy with zero duration; the only purpose being to tie-in the loose ends of the network. The lead time comes before the beginning of any other activity and end time completes the network.

NUMBERING THE NODES

When all the activities have been included in a closed network diagram numbers can be assigned to each node. Each node will be numbered consecutively from the beginning to the end of the network. It is suggested the starting node of the lead time be numbered zero, and the end node of the lead arrow be numbered one.

Numbering should proceed from left to right from lead time to end time. The following steps should be followed when numbering the nodes:

1. Find the initial and final events. The initial event is the one which has arrows emerging from it but none entering it.
2. Number the initial event zero. Generally the initial event should be the first node in the network diagram.
3. Omit all arrows emerging from all numbered nodes. This will create at least one new initial event, which is assigned the next consecutive number. In a case where there is more than one initial event,

consecutive numbers may be assigned as desired. This is continued until numbers have been assigned to all nodes.

The final event is the one which has no arrows emerging from it.

Each activity could be shown as $A_{i,j}$

A = Activity number

i = Number of start of activity A

j = Number of end of activity A

From the following example, the table No. 2 could be made:

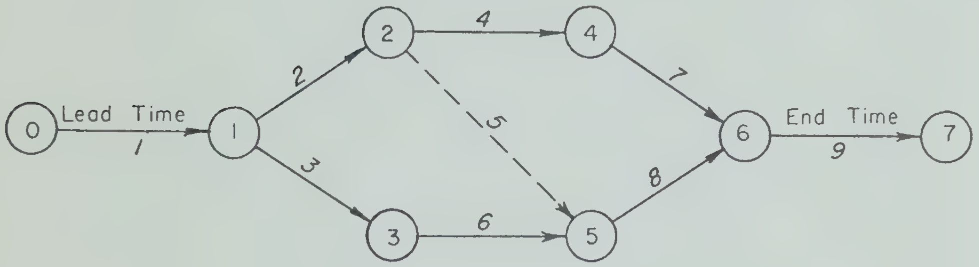


Figure No. 1

TABLE NO. 2 ACTIVITY NUMBERING

ACTIVITY NO. A	START NODE i	END NODE j	SUGGESTED REPRESENTATIVE $A_{i,j}$
1	0	1	$1_{0,1}$
2	1	2	$2_{1,2}$
3	1	3	$3_{1,3}$
4	2	4	$4_{2,4}$
5	2	5	$5_{2,5}$
6	3	5	$6_{3,5}$
7	4	6	$7_{4,6}$
8	5	6	$8_{5,6}$
9	6	7	$9_{6,7}$

The various chains of activities which must be maintained to start the project can be represented by a diagram and the diagram is then a representation of the project. Considerable skill is required in designing a network diagram to satisfy all the requirements imposed by all factors. Figure No. 1 shows a network diagram.

RULES IMPOSED BY COMPUTERS

1. Any two events (nodes) may not be directly connected by more than one activity.
2. Event numbers must not be duplicated in a network.
3. Networks may have only one initial event and only one terminal event.

Rule Number 1 and 2 may be interpreted by means of a portion of a network shown in the following figure (No. 2.)

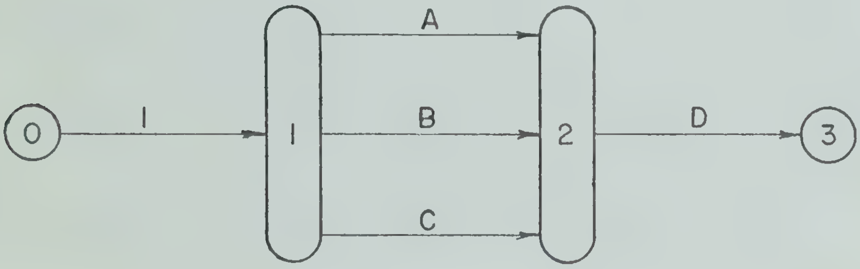


Figure No. 2

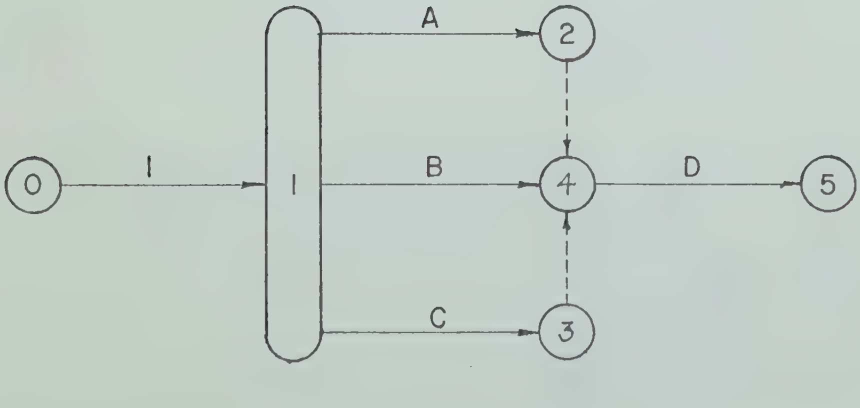


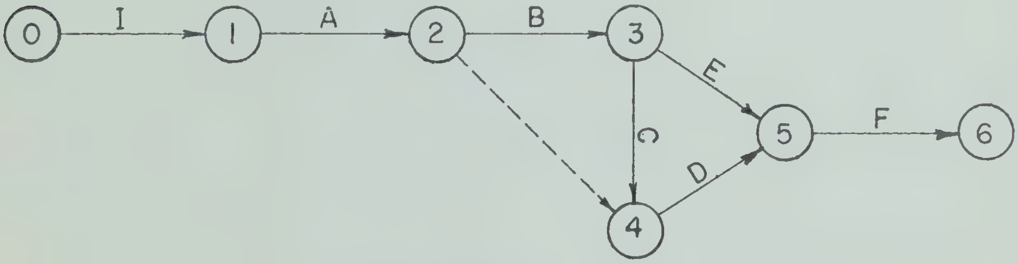
Figure No. 3

According to figure No. 2 this diagram states that before activity D can begin, activities A, B, and C must be completed. This is not intended to imply that activities A, B, and C must be completed simultaneously. They may be done but not necessarily.

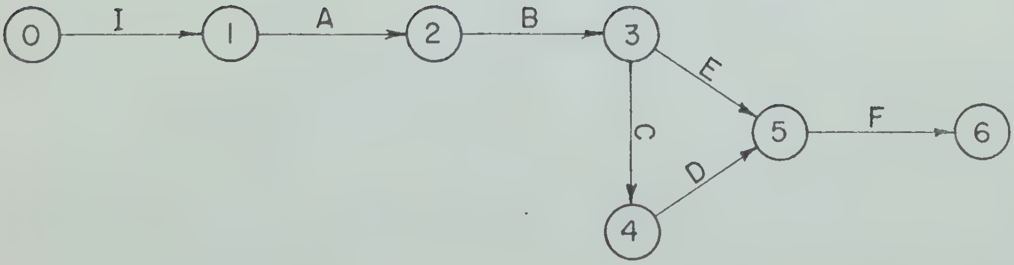
Activities A, B and C may be called duplicate activities. Since the computer could only use event numbers for identification of the units. Therefore, it will not be able to distinguish between the two duplicate activities as indicated in the figure No. 2. One remedy calls for introduction of dummies and events as in figure No. 3.

USING DUMMIES EFFICIENTLY

While the need for dummies in certain cases has been pointed out, it is preferable to avoid unnecessary dummies. For example, consider the diagram in figure No. 4.



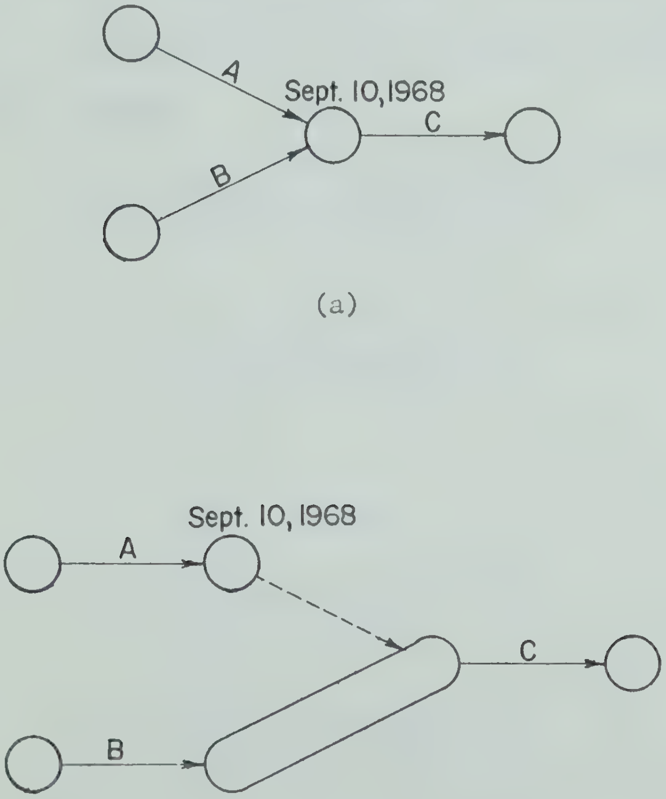
(a) Wrong



(b) Correct

Figure No. 4

Evidently, activity D depends on C, B, and A, but the dependency of D is clear without the dummy 1, 2, which is not necessary. In other cases, it may be desired to introduce dummies for clarity. For example, suppose an important schedule date is noted on a merge point (figure No. 5). Due to the ambiguity of the merge event, it may not be clear what is meant by this date. Does it represent a schedule of completion of A or B, a completion of A and B or the beginning of C? In special cases of this type, it may be desirable to remove the possible misunderstanding by introducing dummies as in figure No. 5.



(a)

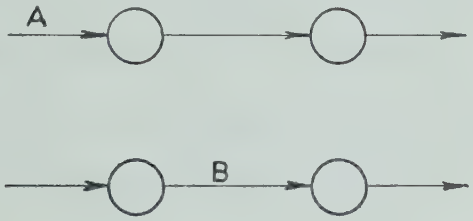
(b)

Figure No. 5

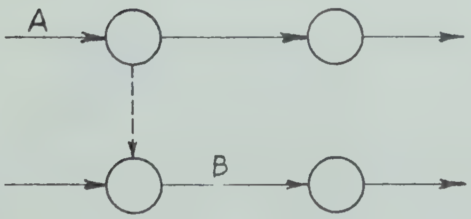
In this example, it is now clear that the date September 10, 1968 refers to completion of activity A, which causes a delay in the beginning of activity C. It is emphasized that this is only an illustration of the flexibility afforded by use of dummies and not a required practice at merge points.

Another important use of dummies is illustrated by the following example:

Suppose that in the portion of a network shown in figure No. 6. It is realized that if activity B depends on activity A because of the commone use of the same resource, such as a skilled person or a piece of equipment. This is not shown in figure; then it may be shown as dependent on A by use of a dummy.



(a) Wrong



(b) Correct

Figure No. 6

EXAMPLE

NOTES

Assuming that a project consists of 9 jobs and one dummy, including lead and end times, number the nodes of the net work diagram in the correct order.

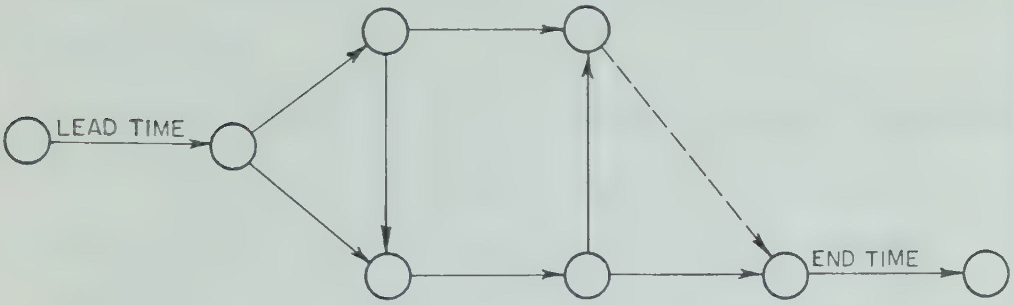


Figure No. 7

SOLUTION

In the above example there are 3 nodes and 8 numbers. Therefore, each activity has two numbers - one indicates the start of an activity and the other the end of the activity. This is shown in figure No. 8.

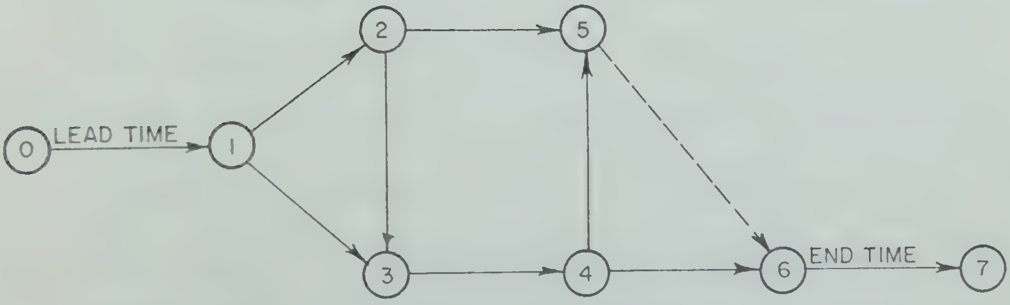


Figure No. 8

In the above example the first node is numbered zero. By deleting all arrows emerging from it, another initial event is created and assigned No. 1. After numbering 1, all arrows emerging from it, will be crossed out and three new initial events are created. Number one of these nodes 2 and so on until all nodes are numbered in like manner.

CRITICAL JOBS

Those jobs which directly control the overall duration of a project are called critical jobs. Any delay in a critical job has a corresponding effect on the project completion. With enough resources, a critical job and hence the project may be completed more quickly. If there are constraints beyond control of the planner, the job durations cannot be shortened. Hence, the critical job will control the overall time of the project.

CRITICAL PATH

A critical path forms a chain of critical jobs running through the network. In any network, there will always be at least one Critical Path or more.

DURATION OF ACTIVITIES

All activities require some time to be completed. This time is called the duration. The duration of each job can be shortened or extended according to the available resources. For example; where an investigation is required for a damsite and enough resources, such as personnel and financing are available it may be possible to complete the

investigation in say 5 years. On the other hand, if resources are limited, it could take possibly 10 years to do the same job. The duration under these conditions would be 5 and 10 years respectively.

NOTES

UTILITY DATA AND TIME COST CURVES

In CPM, each job or activity must have an estimated time or cost magnitude.

Restraints outside of our control may govern the duration and cost of each project. Generally each job has a time-cost curve similar to that shown in figure No. 9 for a drilling job. This curve is referred to as a data curve or direct cost-time curve.

Particular mention should be made of the following significant points on this curve:

1. Least-Time Solution:

   The point showing the shortest possible time for completion of a job and the minimum possible cost for that completion time. Any reduction in time generally requires an increase in resources which entails higher costs.

2. Least-Cost Solution:

   This point indicates the required time for completing a job at the lowest possible direct cost.

3. Optimum Time-Cost Solution:

   This solution is between the above extremes, and it depends on constraints.

4. All Normal Points:

   Points of least-cost and least-time solution are called, All Normal Costs and All Normal Time Points. All Normal Cost is therefore the cost required to do the job in the shortest feasible time. This must cost more because of the added expense of overtime, shift work,

use of more equipment, etc.

NOTES

It should be mentioned that there are more statistical methods for estimation of durations of jobs. Most of the given reference shows these methods in detail.

Assuming that a drilling job requires 600 feet of drilling, each drilling rig is capable of drilling only 100 feet per day, the labour for each drilling rig costs $30 per day and the cost of transportation for each rig is $20, then the curve would be plotted as shown in figure No. 9, and 9A.

TABLE NO. 3 DRILLING UTILITY - COST-TIME CURVE

TIME (days)	NO. OF RIGS	COST OF RENTAL = NO. OF RIGS x DAYS x $150	COST OF TRANSPORTATION NO. OF RIGS x $20/DAY	COST OF LABOUR PER DAY	TOTAL COST OF PROJECT
1	6	6 x 1 x $150 = $900	6 x $20 = $120	1 x 6 x $30 = $180	$1200
2	3	3 x 2 x $150 = $900	3 x $20 = $ 60	2 x 3 x $30 = $180	$1140
3	2	2 x 3 x $150 = $900	2 x $20 = $ 40	3 x 2 x $30 = $180	$1120
6	1	1 x 6 x $150 = $900	1 x $20 = $ 20	6 x 1 x $30 = $180	$1100
7	1	1 x 7 x $150 =$1050	1 x $20 = $ 20	7 x 1 x $30 = $210	$1280

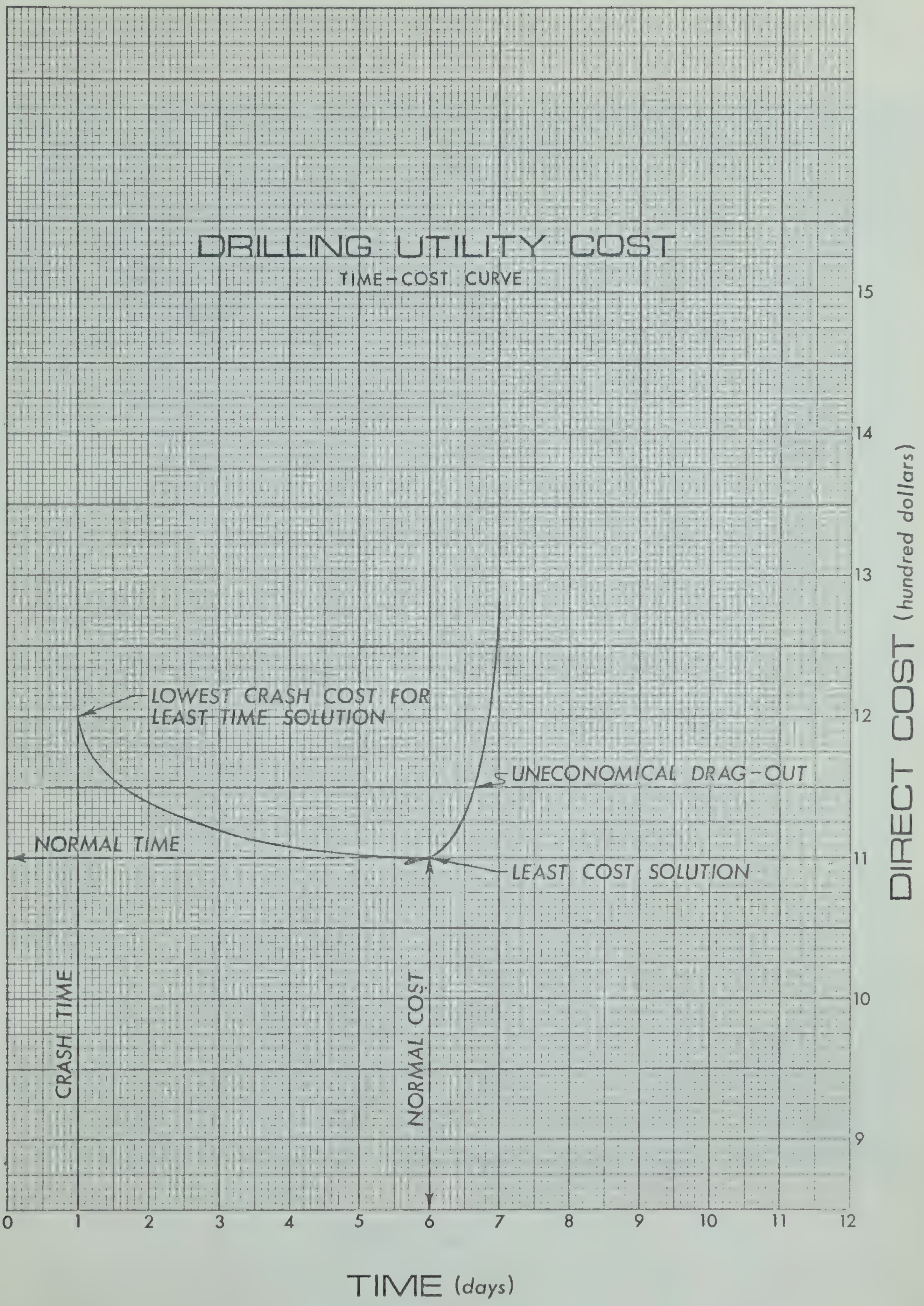

DRILLING UTILITY COST
TIME–COST CURVE
LOWEST CRASH COST FOR LEAST TIME SOLUTION
UNECONOMICAL DRAG-OUT
NORMAL TIME
LEAST COST SOLUTION
CRASH TIME
NORMAL COST
0
1
2
3
4
5
6
7
8
9
10
11
12
TIME (days)
9
10
11
12
13
14
15
DIRECT COST (hundred dollars)

Figure 9

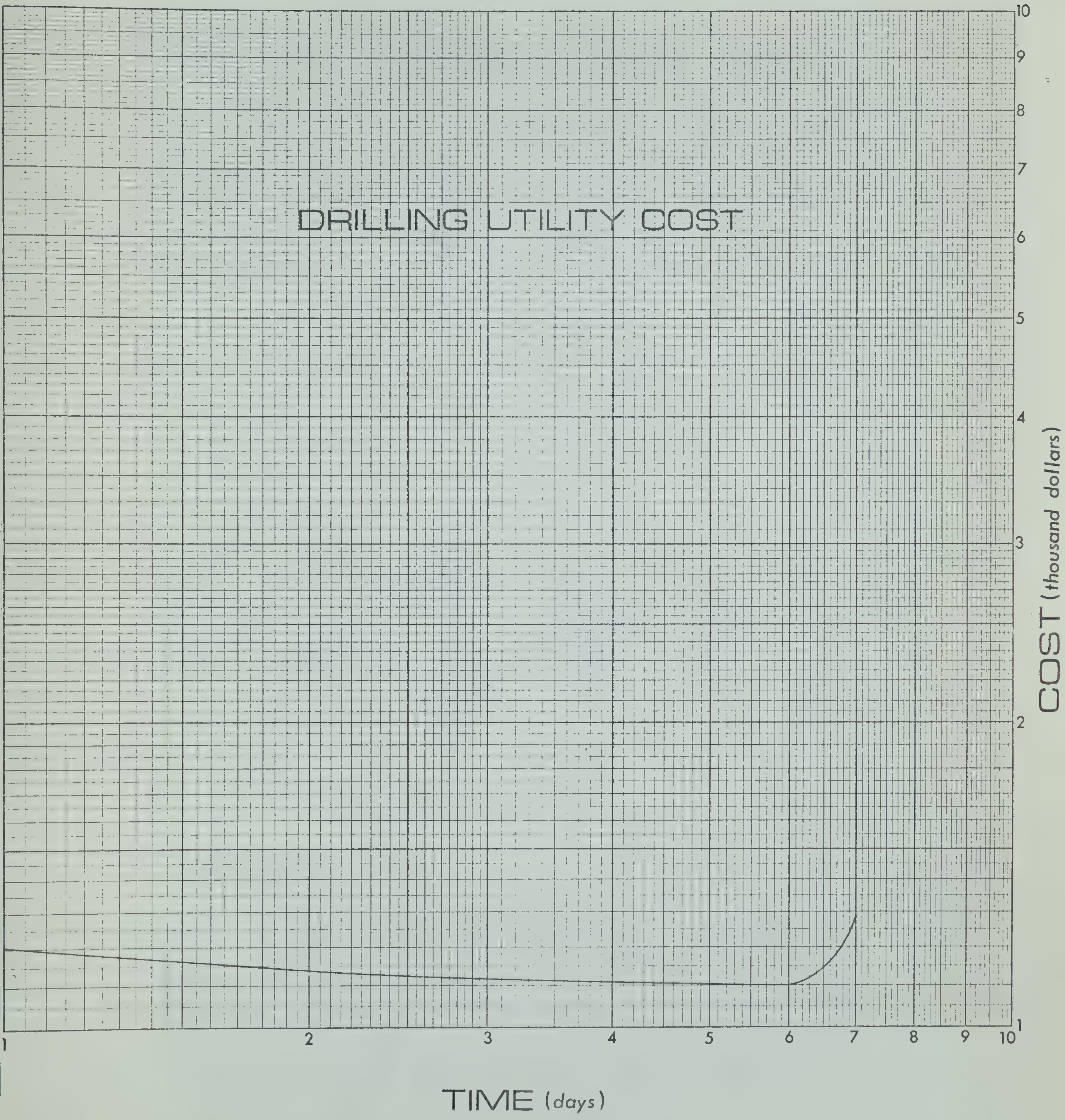

DRILLING UTILITY COST
1
2
3
4
5
6
7
8
9
10
TIME (days)
1
2
3
4
5
6
7
8
9
10
COST (thousand dollars)

Figure 9A

CRITICAL PATH COMPUTATION

NOTES

The following information is required for finding the critical path of a network diagram.

1. EPO - Earliest Possible Occurrence of Events (nodes)

This is the earliest possible time for starting a job (arrow) which begins with a node. The EPO of a node is the end or completion of existing activities. For example, the earliest possible occurrence (EPO) of node 2 in the figure No. 10, is ten days after node 1.

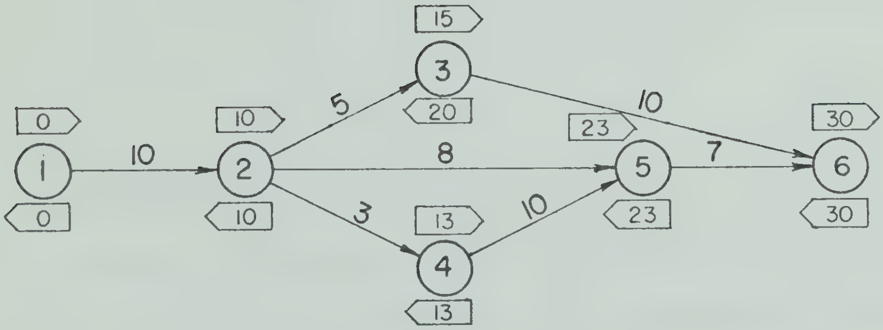


Figure No. 10

This means that jobs 2,3,2,4,& 2,5 can not be started before the end of day 10. To find the EPO of all nodes, assign an EPO of zero to the first node. The EPO for each of the remaining nodes is the maximum summation of the earliest finish times (∝) of all operations that terminate

at that node.

$$EPO = \text{Maximum of } \alpha\text{'s}$$

The EPO of the final node is equated to the earliest possible completion time for the entire project.

2. Earliest Finish Time = α

The earliest finish time for most jobs is the earliest possible time in which that job can be completed. The earliest finish time of the project will therefore be the sum of durations of the longest time path through the network from start to finish. The earliest finish time of a job is then the sum of the EPO of the starting node of the activity plus the duration of the activity. Referring to figure No. 10, the α of the job 2,3 is:

$$\alpha = \text{EPO of node 2} + \text{duration of activity 2,3},$$

or

$$\alpha = 10 + 5 = 15.$$

3. Latest Finish Time = β

This is the latest time an activity could be finished without affecting the overall project duration. The latest finish time of the last activity is controlled by all activities which precede it.

4. Free Float = F_f

The free float of an activity is the difference between its earliest finish time and the earliest possible occurrence of the node at which it terminates. The EPO of the end node minus earliest finish time:

$$F_f = EPO - \alpha$$

Free float of activity 2,5 is EPO of node 5 minus earliest finish time of activity 2,5 from figure No. 10.

$$F_f = 23 - 18 = 5 \text{ days}$$

This leeway associated with activity 2,5 may be used up in a delay of as much as 5 days in the completion of activity 2,5 but this will not affect or delay activities emerging from node 5. It is obvious that no activity with a free float can control the duration of an entire project. The completion of an activity can be advanced as much as the Free float or start of the activity can be delayed by an amount equal to its free float without affecting the EPO of any node. This means that the EPO of the last node will not be affected and hence the project duration will not be altered. This could then limit the search for "critical activities" to those that have zero free float. However, all activities with zero free float do not necessarily control the project duration, therefore, they are not on the "critical" route. Only activities which have zero free float and also form a continuous chain starting at the first node and ending with the last node are on the critical path.

5. Latest Possible Occurrence LPO

The LPO of each node is the latest possible time that all of the activities that terminate at that node can finish without causing the duration of the project to exceed the value originally calculated with the earliest possible occurrence. The latest possible occurrence of the last node is equal to the earliest possible occurrence of that node.

6. Latest Start Time = δ

This is the latest time that an activity can start without affecting the total project duration. This is the latest possible occurrence of the node at which the activity terminates minus its duration.

δ = LPO of Terminating Node - t

where

LPO = latest possible occurrence

t = duration.

NOTES

For example, in figure No. 10, the last node has an:

LPO = EPO = 30 days.

The δ of Activity 3,6 is:

δ = LPO - duration of activity 3,6.

δ = 30 - 10 = 20 days.

The LPO of a node is the minimum of all δ's of the activities that originate there. For example, the LPO of node 2 figure No. 10 is the minimum of 10 and 15 which is:

δ = 23 - 8 = 15 for activity 2,5

δ = 13 - 3 = 10 for activity 2,4

δ = 20 - 5 = 15 for activity 2,3

δ = 10 is selected as LPO for node 2.

7. Total Float = T_f

The full amount of time by which the start of an activity may be delayed without causing the project to last longer. The difference between the earliest start time and latest start time is called total float.

$T_f = \delta - \gamma$

where T_f = total float

δ = latest start time of activity

γ = earliest start time of same activity.

8. Interfering Float

Total float minus Free float

9. Earliest Start Time = γ

Earliest start time is equal to EPO of start node.

NOTES

This may cause delay in some succeeding activities but will not retard the total project completion.

SUMMARY

The following steps are involved in CPM computation:

1. Define project.
2. Break the project into its activities.
3. Draw an arrow diagram that defines the sequence of performing these activities.
4. Estimate duration of each activity.
5. Calculate EPO of each node.
6. Calculate α = earliest finish time.
7. Calculate β = latest finish time.
8. Calculate F_f = free float.
9. Calculate T_f = total float.

*10. Calculate I_f = interfering float.

11. Select critical route through critical jobs.

From the network diagram and critical path, one could answer the following questions:

(a) How long it takes to complete the entire project (completion time)?

(b) Which jobs are controlling completion time?

(c) How much leeway is in the operation that does not control the completion time?

(d) Is it possible to delay any jobs for resources allocation?

CRITICAL PATH METHOD COMPUTATION FORMULAE

The following formulae are useful in critical path method computation:

α = earliest finish time

β = latest finish time = LPO of end node

γ = earliest start time = EPO of starting node

δ = latest start time

t = duration

T_f = total float

F_f = free float

I_f = interfering float

$$\gamma = \alpha - t \tag{1}$$

$$\delta = \beta - t \tag{2}$$

$$T_f = \beta - \alpha = \delta - \gamma \tag{3}$$

$$F_f = \gamma \text{ of end node } - \alpha \text{ of activity in question} \tag{4}$$

$$I_f = T_f - F_f \tag{5}$$

$$\alpha = \text{EPO of start node} + t \tag{6}$$

$$\text{EPO} = \gamma \tag{7}$$

$$\text{LPO} = \beta \tag{8}$$

ALLOCATION OF RESOURCES

NOTES

After the network has been drawn before the final CPM calculations have been performed, the network should be analysed and evaluated. The examination of the results not only serve as a check but also discloses possible ways of improving the proposed project plan, schedule and management. The results should be checked mainly against available resources. As it was discussed in previous lectures, the project should be broken down into its elements and then the fundamental sequencing decisions can be quickly resolved by the planner. This could be done by satisfying three logical, sequential questions. The sequencing that is controlled by time, space, and technology requirements of a project is not subject to arbitrary decision making on the part of the planner. However, after a network diagram is finished, it must be examined by a group of skilled planners. The network should actually be criticized, questioned and the questions must be answered to assure a logical network diagram. Some sequential relationships are controlled by resources such as:

1. Manpower
2. Equipment
3. Funds

These resources and their availability should be carefully reviewed during network analysis. A list of available men and their skills should be prepared. Groups of men who are performing various tasks should be studied. The average performance rate of each man should be statistically

computed. For example, prior to planning, scheduling or time duration estimates the rate of performance should be calculated. If the jobs are water resources project planning, soils, foundation or hydrology studies, the rate of performing each task should be estimated, and the available manpower for each of these jobs should be indicated with a view to a possibility of future expansion or a decrease in manpower resources. The same analysis should be made for all other related resources such as, drilling equipment, surveying vehicles, and instruments, as well as office space and funds.

Generally, there are two types of sequencing decisions that concern the planner with resource allocation:

1. The first could be handled most conveniently during construction of the network diagram. These sequencial decisions are made before calculations for the critical path analysis. For example, consider the following case: In excavation of two diversion tunnels for a dam, after answering the above sequential set of questions, it seems that both tunnels could be driven concurrently following necessary surveying, that is to say as far as time, space, men and technology requirements for the project are concerned. Figure No. 11 is the correct network diagram for this example.

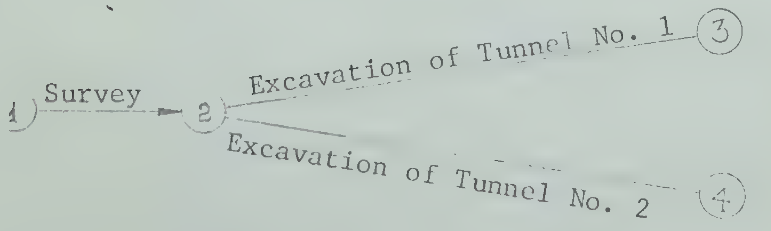
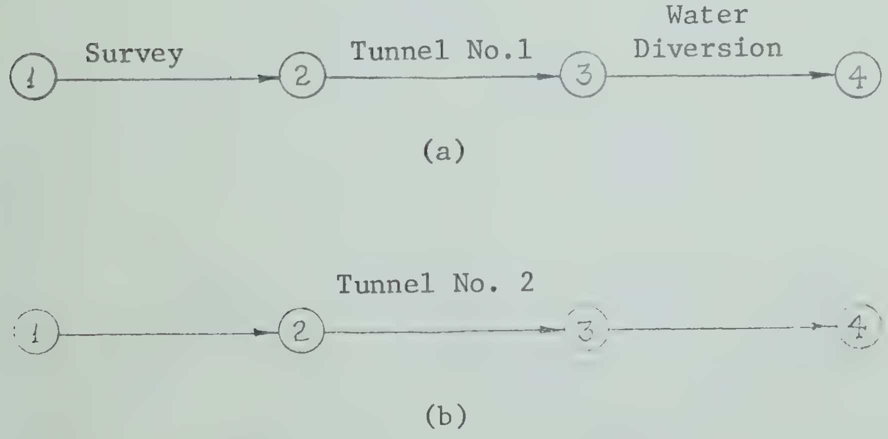


Figure No. 11

This network requires that one machine be available on site for the excavation of each tunnel. Should two machines be used so that activities 2,3 and 2,4 can be done concurrently? The answer to this question involves an evaluation concerning the entire project and the total effect of using two machine or one should be evaluated. If the Department of Water Resources has access to only one machine then how should the activity be sequenced? Clearly the operations could not be performed as indicated in the first network diagram. It then becomes necessary to make a second network diagram which suits the constraint, that is the equipment constraint. The two tunnels are necessary duplicates but are not really alternatives. With only one machine available, each of the networks in figure No. 12 is satisfactory and neither is preferable.



(a)

(b)

Figure No. 12

The foregoing is an example of resource allocation. It is noted that no calculations are necessary to determine the best sequence of operations requiring common resources. When two operations require the same common resource, and each operation is followed by an identical chain of operations which terminate at a common node, there is a conflict of resources and an adjustment is, therefore, necessary to establish resource allocation.

2. The second type of sequencial decision is necessary when the possibility of a resource conflict is not readily apparent. The consequences of any given sequencing can not be determined until after the network diagram is completed and calculations have been made. For example, assume that operations 2,8 and 4,5 in figure No. 13 require a common resource such as a hydraulic engineer, an economist or an agrologist.

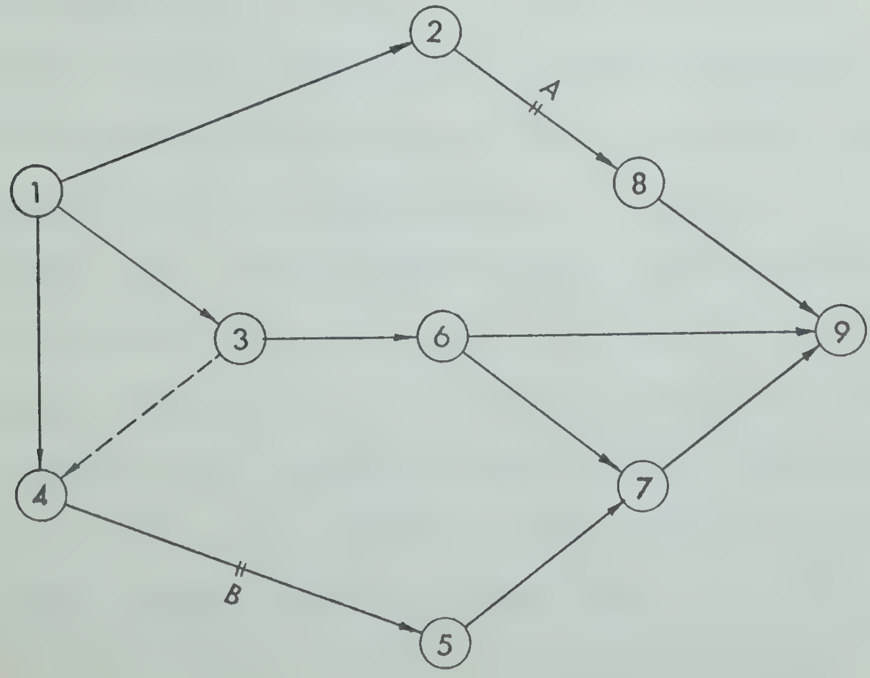


Figure No. 13

NOTES

By looking at the above network diagram one can not be certain that a resource conflict exists. It would be risky to make a decision regarding sequencing of A and B before the operation times have been estimated and the float has been computed. After all calculations have been completed, it can then be determined whether a resource conflict exists between A and B. If a conflict does exist, a critical path is required in order to determine which operation must be performed first and what its effect will be on the project, and where necessary correct the sequence. The case arises when the operations requiring a common resource do not depend on a common operation and are not followed by an identical chain of operations. The resources should be used continously where necessary and should not be idle. All resources must be carefully scheduled. Idle resources are indicators of poor management, confused planning, or both.

Any plan for constructing a project must be one which utilizes that number of men and pieces of equipment which are available to management for use on the project. For example: If a network diagram suggests that four engineers are required on the same project for a period of one week, and management has only two engineers, the diagram becomes meaningless since management cannot allocate four engineers but only two. This example illustrates the first goal of network analysis for resource allocation.

To determine the levels of resources, which must be scheduled in order to implement the plan or project, the second goal of this analysis is to alter an unacceptable CPM plan, into an efficient useable CPM plan. This is called resequencing of the operations or it could be called resource scheduling. Information required for research scheduling are as follows:

1. The project diagram or network diagram resulting from answering the sequential questions, as indicated in previous lectures must be prepared neglecting resource availability.

2. Associated with each operation, and its duration, and the amount of each resources required to maintain this duration should be estimated. It is to be noted that in the resouce scheduling method for all of them, the maximum level of available resources are assumed to be constant for the duration of project. Lets say two engineers are always available rather than during only the first two months of the project. If the maximum amounts of resource availability changes during the performance or planning, then updating must be done. The maximum available resources must at least equal to the amount of that resource required by a single operation of the project. Resources scheduling method is scheduling of work in such a way that it does not require more of a resource than can be supplied by the management at any one time.

In other words, it provides a network diagram which does not need any excess of chief resource. Whenever the availability of a resource is less than the maximum requirement in a network diagram some choice in allocating the resource should be made. Unless a random choice is made, some criterian is needed by which we may be directed toward the most efficient allocation of the resource, the criterian used is reflected in Free Float slack. By delaying a job with a large Free Float, it tends to absorb some of the slacks and thereby increases the utilization of the resources consumed by that job. If two or more jobs with the same slack are competing for the same resource then the duration may be used as a secondary criterian. The job with the shortest duration should be chosen because it will give the earliest opportunity for reassessing the situation. If the competing jobs have the same duration as well as the same slack the resource allocation may be made at random. There is little or no advantage in having a third criterian. The total float for a job is used to infer a priority but it is worth knowing that the policy decision is also implied.

MATRIX ANALYSIS OF NETWORK DIAGRAM

NOTES

Graphical network analysis and calculation of Critical Path could have some errors. In an attempt to eliminate these errors and make CPM calculations easier, a matrix of activities and events has been used with success. Any network can be represented by a table or a matrix. For example, the network diagram in figure No. 14 could become the matrix in table No. 4. Supposing a network diagram has a total number of N events. It is possible to make a matrix consisting of N columns and N events. Referring to table No. 4, each entry in the matrix corresponds to the connections between the initial and final events of an activity and the value of the entry is the duration of that activity. Some of the cells in the matrix will not be used since an event can not be followed by itself nor by one which has a lower number. This is shown in table No. 4 by the unused part of the matrix being left blank. Any entry which falls into the blank part will indicate incorrect numbering of the events. After making the matrix and entering the duration of each activity in the correct column and rows, the matrix form is completed. By adding another column and another row, the earliest possible occurrence time of events, and the latest possible occurrence time of the events can also be calculated as shown in table No. 4.

EARLIEST POSSIBLE OCCURRENCE TIME EPO

Since the earliest possible occurrence of the first node is zero, enter zero under column, "earliest possible occurrence time" (EPO) of the events, in the first row. This indicates the earliest possible occurrence or EPO of the first node. Earliest possible occurrence time for node No. 2 will be the duration of activity ($A_{1,2}$) plus earliest possible occurrence time of node No. 1 which was zero, therefore:

$$(EPO)_2 = 0 + 1 = 1$$

Enter this number as earliest possible occurrence time of node No. 2. Under column No. 3, there is also only one entry indicating a duration of 6 for activity ($A_{2,3}$). To calculate earliest possible occurrence time for node No. 3, the duration of activity ($A_{2,3}$) will be added to earliest possible occurrence time of node No. 2, therefore:

$$(EPO)_3 = 1 + 6 = 7$$

for node No. 3. Under column No. 4, there are two entries: 8 and 3. This means there are two activities terminating at node No. 4. Calculate the earliest possible occurrence for this node. As it was stated earlier, the maximum earliest possible occurrence time will be selected, therefore:

$$(EPO)_4 = 8 + 1 = 9$$

$$(EPO)_4 = 7 + 3 = 10$$

and 10 will be selected as earliest possible occurrence time for node No. 4. Under column No. 5, there are two entries: 7 and 3. The earliest possible occurrence for this node will be the maximum of (7 + 7) and (3 + 10).

TABLE No. 4

MATRIX FORM OF NETWORK ANALYSIS

i	j								Activity Ai,j	EARLIEST		LATEST		FLOAT			
	1	2	3	4	5	6	EPO	LPO		Start $\gamma = \alpha - t$	Finish α = EPO + t	Start δ = LPO − t	Finish $\beta = \delta + t$	Total (T_f) $= \beta - \alpha$	Free F_f=EPO $- \alpha$	Interfering $(I_f)=T_f-F_f$	REMARKS
	COLUMN						(1)	(2)	(3)	(4=5−t)	(5=1+t)	(6=2−t)	(7=6+t)	(8=7−5)	(9=1−5)	(10=8−9)	
1		1					0	0	1,2	0	1	0	1	0	0	0	Critical
2			6	8			I	1	2,3	1	7	1	7	0	0	0	Critical
									2,4	1	9	3	11	2	1	1	
3				3	7		7	7	3,4	7	10	8	11	1	0	1	
									3,5	7	14	7	14	0	0	0	Critical
4					3		10	11	4,5	10	13	11	14	1	1	0	
5						1	14	14	5,6	14	15	14	15	0	0	0	Critical
6							15	15									
LPO	0	1	7	11	14	15											

ABBREVIATIONS

EPO = Earliest possible occurrence time.
LPO = Latest possible occurrence time.
i = Initial event of activity.
j = End event of activity.
t = Duration
I_f = Interfering float

α = Earliest finish time
β = Latest finish time
γ = Earliest start time
δ = Latest start time
T_f = Total float
F_f = Free float

Therefore, 14 will be selected as the earliest possible occurrence time for node No. 5. Under column 6, there is only one entry, which indicates the duration of activity ($A_{5,6}$). Add this activity duration to the earliest possible occurrence time of node No. 5, which was 14 and the earliest possible occurrence time of 15 for node No. 6 will be obtained.

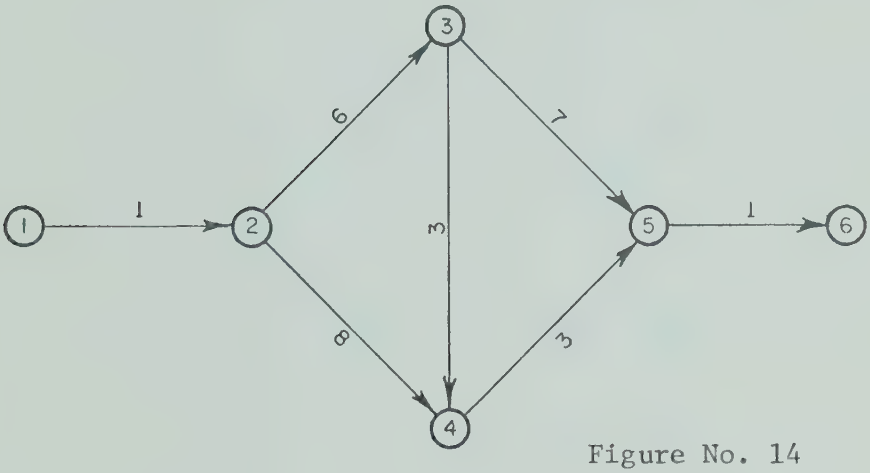


Figure No. 14

NETWORK DIAGRAM USED FOR MATRIX TABLE

LATEST POSSIBLE OCCURRENCE TIME BY MATRIX METHODS

At the bottom of table No. 4, another row was added for the latest possible occurrence time of the events. Starting with node No. 6 which is the last node in the network diagram. Since it has already been stated that the earliest possible occurrence time and latest possible occurrence time for the last node are the same, therefore No. 15, which was the earliest possible occurrence time for node No. 6 will also be the latest possible occurrence time. To calculate latest possible occurrence time for node No. 5, referring to the table No.4, it can be noted that there is only one entry opposite node No. 5, therefore, substracting this duration from 15, which was latest possible occurrence time for node No. 6, the latest possible occurrence time of 14 will be obtained for node No. 5.

Node No. 4 also has only one entry, substracting the duration of activity ($A_{4,5}$) from latest possible occurrence time of event No. 5 which was 14, the latest possible occurrence time for node No. 4 will be obtained which is equal to 11. Enter 11 under column No. 4. Opposite node No. 3 there are two entries, 3 and 7, therefore, in this case, there are two latest possible occurrence times for event No. 3. One is from activity ($A_{3,4}$), one is from activity ($A_{3,5}$). Latest possible occurrence time for event No. 3 through activity ($A_{3,5}$) will be 14 - 7 which makes 7, and through activity ($A_{3,4}$) will be 11 - 3 which makes 8. Since the latest possible occurrence time is the

NOTES

minimum of latest possible occurrence times, therefore, 7 will be selected and will be entered under column No. 3 indicating latest possible occurrence time for node No. 3. Opposite node No. 2 there are also two entries, 6 and 8, under columns 3 and 4, respectively. In this case, also there are two latest possible occurrence times, one through activity (2,4):

LPO = 11 - 8 = 3

and another through activity ($A_{2,3}$):

LPO = 7 - 6 = 1

therefore, the minimum will be selected which is 1. Enter one under column No. 2. Activity ($A_{1,2}$) has one entry with duration of one, therefore, 1 - 1 = 0 which is the latest possible occurrence time for node No. 1. After completion of latest possible occurrence time and earliest possible occurrence time it will be possible to continue with the calculation of other required information for critical path computations. For ease of calculations, LPO has been repeated in a vertical column following EPO.

To continue calculations, all activities must be entered in another column, and added to table No. 4. Enter activities composed of rows and columns under this column. From node No. 2, two activities are emerging and it will be noted that they are ($A_{2,3}$) and ($A_{2,4}$). These activities also will be entered under activity column. Node No. 3 has also two entries, ($A_{3,4}$) and ($A_{3,5}$). This will continue until table is completed.

CALCULATION OF EARLIEST FINISH TIME OF ACTIVITIES (α)

As it was stated, earliest finish time of each activity is equal to earliest possible occurrence of the node which this activity emerges from plus activity duration. For example, activity ($A_{1,2}$) has a duration of one and earliest possible occurrence of node No. 1, which is the start of this activity is zero, therefore, earliest finish time for activity ($A_{1,2}$):

$$\alpha_{1,2} = 0 + 1 = 1$$

This is the same thing with activity ($A_{2,3}$).

In this case, duration of activity ($A_{2,3}$) is six and earliest possible occurrence of node No. 1 is one, therefore, earliest finish time for this activity will be 6 + 1 which makes 7. Enter 7 opposite activity ($A_{2,3}$). Activity ($A_{2,4}$) has a duration of eight and earliest possible occurrence of node No. 2 is one, therefore, 8 + 1 makes 9. Enter 9 opposite activity ($A_{2,4}$); use this same process for all other activities.

CALCULATIONS OF LATEST START TIME OF EACH ACTIVITY (σ)

Latest start time of each activity is equal to the difference between duration of activity and latest possible occurrence of the node which this activity terminates at. For example, the latest start time of activity ($A_{1,2}$) will be the result of subtraction of duration ($A_{1,2}$) from latest possible occurrence of node No. 2:

$$\sigma_{1,2} = 1 - 1 = 0$$

the latest start of activity ($A_{2,3}$) will be latest possible occurrence of node No. 3 which is 7 minus duration of

activity ($A_{2,3}$) which is 6:

$$\delta_{2,3} - 7 - 6 = 1$$

This calculation will be continued until all the latest start time of all activities have been computed.

LATEST FINISH TIME OF ACTIVITIES (β)

The latest finish time of activities is equal to latest start of that activity plus duration. For example, the latest finish time for activity ($A_{1,2}$) is the latest start time of activity ($A_{1,2}$) plus duration of activity ($A_{1,2}$). Therefore:

$$\beta_{1,2} = 0 + 1 = 1$$

The latest finish time for activity ($A_{2,3}$) is the latest start for this activity plus the duration for activity 2,3 or 6. Therefore:

$$\beta_{2,3} = 1 + 6 = 7$$

This calculation will be continued until all the activities have the latest finish time.

EARLIEST START TIME OF ACTIVITIES (γ)

The earliest start time of activities is equal to the earliest finish time of the activity previously calculated, minus the duration of this activity. For example, the earliest start of activity ($A_{1,2}$) is equal to the earliest finish time of this activity, minus its duration. Therefore, the earliest start time for activity ($A_{1,2}$) will be 0. The same way with activity ($A_{2,3}$); the earliest finish time for this activity is 7, the duration for this activity is 6, therefore, the difference is 1, which indicates the earliest

NOTES

start time for activity 2,3. This computation will continue until all the earliest start times for all activities have been calculated.

TOTAL FLOAT CALCULATION (T_f)

Total slack or total float is equal to the difference between the latest finish time of activity minus earliest finish of that activity. For example, total float of activity ($A_{1,2}$) is equal to latest finish time of that activity which is one minus the earliest finish of that activity which is also one. Then:

$$T_f = 1 - 1 = 0$$

the total float for this activity will be 0. The same with activity ($A4_{,5}$). In this activity, the latest finish time is 14, the earliest finish time is 13, the difference will be one, therefore, the total float for this activity will be one.

FREE FLOAT (F_f)

Free float or free slack is the difference between the earliest finish time of that activity and the earliest possible occurrence time of the node which this activity terminates at, for example, free float of activity ($A_{1,2}$) will be the difference between the earliest finish time of this activity which is one, and the earliest possible occurrence of node No. 2, which is also one. Therefore, free float will be 0. Free float of activity ($A_{4,5}$) will be the difference between 13 which is the earliest finish time for this activity and 14 which the earliest possible

NOTES

occurrence time of node No. 5, therefore, free float for this activity will be one. With the same procedure, all free float of the activities will be calculated and entered in correct columns.

INTERFERING FLOAT

Interfering float is the difference between total float and free float, therefore, simply take the difference between these two floats and enter in the column designated as interfering float.

CRITICAL PATH

The critical acitivites are all the activities which have a zero free float and also a zero total float and a zero interfering float. Therefore in this example, the critical activities are activity 1,2, activity 2,3, activity 3,5 and 5,6, which have total float, free float and interfering float equal to zero.

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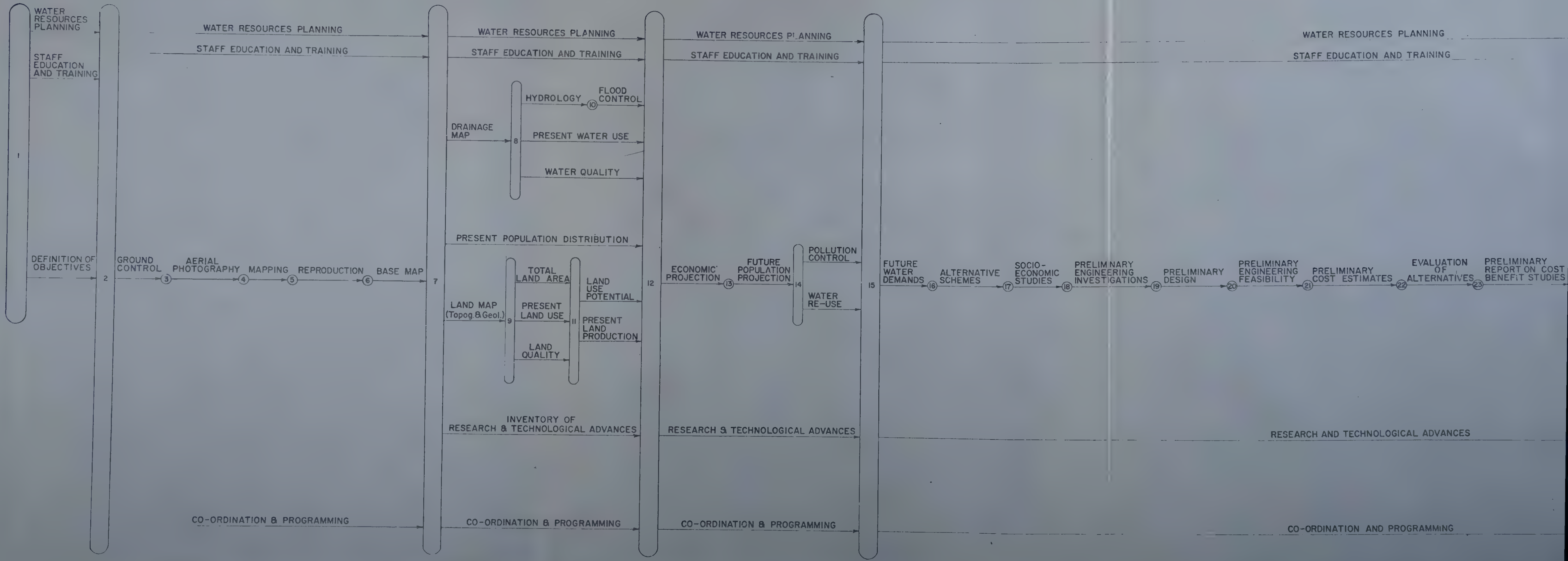

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PRELIMINARY DESIGN
⑳
PRELIMINARY ENGINEERING FEASIBILITY
㉑
PRELIMINARY COST ESTIMATES
㉒
EVALUATION OF ALTERNATIVES
㉓
PRELIMINARY REPORT ON COST BENEFIT STUDIES
RESEARCH AND TECHNOLOGICAL ADVANCES
CO-ORDINATION AND PROGRAMMING

ATER RESOURCES
ETWORK FLOW DIAGRAM

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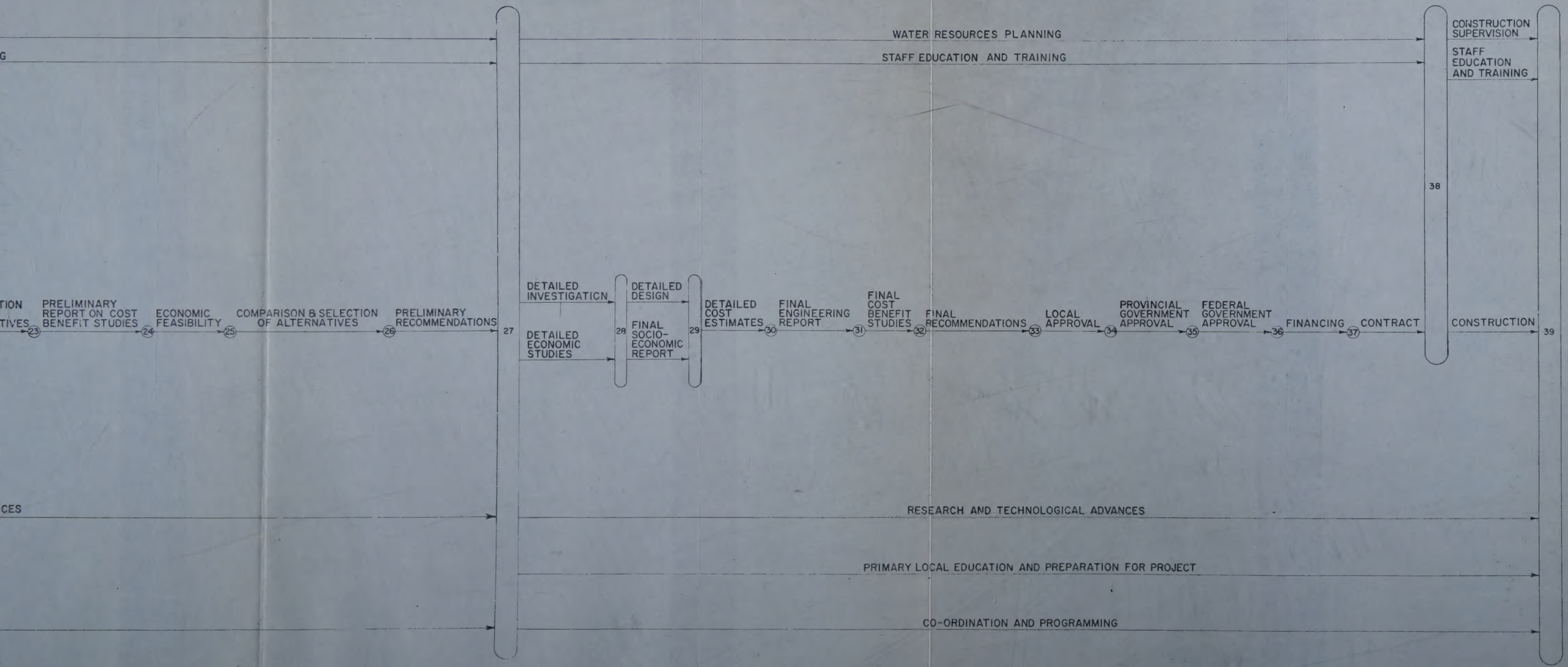

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CES
RESEARCH AND TECHNOLOGICAL ADVANCES
PRIMARY LOCAL EDUCATION AND PREPARATION FOR PROJECT
CO-ORDINATION AND PROGRAMMING

LEGEND

SYMBOL	NAME	DESIGNATION
→	ARROW	ACTIVITY (JOB)
⬯ ○	NODES	START OR END OF ACTIVITY
- - - →	DUMMY	INDICATES ONLY DIRECTION OF FLOW

NOTE:

THIS NETWORK DIAGRAM HAS BEEN SIMPLIFIED AND ALL DUMMIES HAVE BEEN ELIMINATED THEREFORE THIS NETWORK DIAGRAM CANNOT BE USED FOR ANY COMPUTER PROGRAMMING.